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On the Electric Charge of Monopoles at Finite Temperature

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Abstract

We calculate the electric charge at finite temperature T for non-Abelian monopoles in spontaneously broken gauge theories with a CP violating θ-term. A careful treatment of dyon's gauge degrees of freedom shows that Witten formula for the dyon charge at $T = 0$, $Q = e(n - \theta/2\pi)$, remains valid at $T \neq 0$.

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Whenever CP invariance is violated by a θ-term, dyons acquire an electric charge which depends on to the vacuum angle θ through the remarkable relation [1]:

$$Q = -\frac{e\theta}{2\pi} + ne \tag{1}$$

Here e is the unit electric charge and n an integer. This relation was originally obtained, starting from an $SO(3)$ gauge theory spontaneously broken to $U(1)$ by the vacuum expectation value of a Higgs field in the adjoint representation, using semiclassical arguments and also by canonical methods. Following this last approach, one defines the operator N that generates gauge transformations around the $U(1)$ (electromagnetic) surviving symmetry and then imposes, as an operator statement,

$$\exp(2\pi i N) = I \tag{2}$$

Now, when a CP violating term of the form

$$\Delta\mathcal{L} = \theta \frac{e^2}{32\pi^2} {}^*F_{\mu\nu}F^{\mu\nu} \tag{3}$$

is added to the original Lagrangian, one can see that condition (2) implies relation (1).

Afterwards there have been several investigations of fermion-monopole systems leading to the determination of the monopole quantum numbers. According to different assumptions on boundary conditions for the Dirac equation, different results were obtained for the fermion contribution to the monopole electric charge [2]-[3].

Gauge invariance and topological considerations are the basic ingredients for deriving formula (1) much in the same spirit as they are in the arguments leading to quantization of the Chern-Simons coefficient both at the classical and quantum level [4]-[6]. It is then natural to expect that eq.(1) does not get smoothly renormalized at finite temperature. An analysis of an abelian fermion-Dirac monopole system at finite temperature reported in these pages [7] seems to indicate, however, that at finite temperature the dyon's charge is no more quantized according to eq.(1). As the authors already remarked, their analysis is connected with that leading to a temperature dependent coefficient for the Chern-Simons effective action which arises when considering three-dimensional fermions. Now, it has been shown in refs.[8]-[10] that the

requirement of gauge invariance together with topological reasons impose severe constraints on the temperature dependence of the induced charge in this last system: it can be at most an integer function of the temperature so that any smooth behavior of the induced charge with the temperature is excluded. One can then think that results of [7] indicating a temperature dependent change in formula (1) are the reflection of the above mentionned subtleties already encountered at $T = 0$ for fermion monopole systems as well as those arising in the analysis of Chern-Simons theories at finite temperature.

It is then an interesting open question, already stressed in [7], whether formula (1), which is known to hold for non-Abelian monopoles in spontaneously broken gauge theories with a CP violating term (and no fermions), is modified at finite temperature. It is the purpose of the present work to discuss this issue showing that a careful treatment of gauge degrees of freedom leads to condition (1) both at $T = 0$ and at $T \neq 0$.

We start for definiteness with an $SO(3)$ gauge theory spontaneously broken to $U(1)$ by the vaccum expectation value of a Higgs field $\vec{\phi}$ in the adjoint representation. The corresponding Lagrangian reads

$$\mathcal{L} = -\frac{1}{4}\vec{F}_{\mu\nu}^2 + \frac{1}{2}(D_\mu\vec{\phi})^2 - V[|\vec{\phi}|] \tag{4}$$

here $V[|\vec{\phi}|]$ is a symmetry breaking potential having its minima at $\vec{\phi} = \vec{\phi}_0$ and

$$D_\mu\vec{\phi} = \partial_\mu\vec{\phi} + e\vec{A}_\mu \wedge \vec{\phi} \tag{5}$$

As it is well known this Lagrangian admits monopole-like solutions [11]-[12] with quantized magnetic charge and, at the classical level, arbitrary electric charged [13]. At the quantum level, it has been proven that the dyon electric charge Q is quantized to be an integer multiple of the fundamental charge e, $Q = ne$ [14]. As stated above, the situation radically changes when a CP violating term of the form (3) is added to Lagrangian (4)

$$L = \mathcal{L} + \frac{\theta e^2}{32\pi^2}{}^*\vec{F}^{\mu\nu}.\vec{F}_{\mu\nu} \tag{6}$$

with ${}^*F^{\mu\nu} = (1/2)\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$. It is precisely the finite temperature behavior of the system with dynamics governed by Lagrangian (6) that we shall study in what follows.

As stressed in ref.[2] the electric degree of freedom of the dyon corresponds essentially to a rigid rotator collective coordinate associated with gauge transformations. Quantization of the dyon charge will then result from quantization of the rigid rotator energy levels. To see this in more detail let us recall that the static, spherically symmetric 't Hooft-Polyakov monopole solution can be written in the form

$$\vec{A}_i^{mon}(\vec{x}) = \frac{1}{e}(K(r)-1)\vec{\Omega}\wedge\partial_i\vec{\Omega} \tag{7}$$

$$\vec{\phi}^{mon}(\vec{x}) = \frac{1}{e}\frac{H(r)}{r}\vec{\Omega} \tag{8}$$

Here the isovector $\vec{\Omega}$ is defined as

$$\vec{\Omega} = \frac{\vec{x}}{r} \tag{9}$$

and $H(r)$ and $K(r)$ satisfy at the origin $H(0)=0$, $K(0)=1$ while at infinity one has $H(r)/r|_\infty = |\vec{\phi}_0|$, $K(r)|_\infty = 0$ ensuring that the monopole has unit magnetic charge.

The monopole can be endowed with electric charge by including an $\vec{A}_0$ component [13],

$$\vec{A}_0(\vec{x}) = \frac{1}{er}J(r)\vec{\Omega} \tag{10}$$

with $J(0)=0$ and $J(r)\to Mr+b$ for $r\to\infty$. (M, which has the dimension of a mass, sets the scale for J while b determines the electric charge of the dyon). Indeed, the $U(1)$ electromagnetic field, as identified by 't Hooft [11],

$$\mathcal{F}_{\mu\nu} = \vec{\Omega}.\vec{F}_{\mu\nu} - \frac{1}{e|\vec{\phi}|^2}\vec{\Omega}.D_\mu\vec{\phi}\wedge D_\nu\vec{\phi} \tag{11}$$

shows the existence of a radial electric field

$$\mathcal{F}_{0r} = -\frac{d}{dr}\left[\frac{J(r)}{er}\right] \tag{12}$$

and an electric charge

$$Q \equiv \int dS_i\mathcal{F}_{0i} = \frac{4\pi b}{e} \tag{13}$$

The coupled Euler-Lagrange equations relate b to the other parameters of the theory in such a way that the electric charge remains unquantized at the classical level.

There is an alternative way of obtaining electrically charged monopoles from 't Hooft-Polyakov solution, which turns to be more convenient concerning quantization. Indeed, working in the $\vec{A}_0 = 0$ gauge, one can make arise the dyon degree of freedom [2] by considering time-dependent transformations $U(\vec{x}, t)$, leaving the Higgs field invariant, of the form

$$U[\lambda] = \exp[i\lambda(r,t)\vec{\Omega}.\vec{t}] \tag{14}$$

which act on the $A_i's$ so that

$$A_i^{mon} \to U[\lambda]A_i^{mon}U^{-1}[\lambda] + \frac{i}{e}U[\lambda]\partial_i U^{-1}[\lambda] \tag{15}$$

Here t^a $(a = 1, 2, 3)$ are the $SO(3)$ generators, $t^a = \sigma^a/2$, and we write $A_i = \vec{A}_i.\vec{t}$. Because of the underlying spherical symmetry, λ depends only on the radial coordinate r. Note that U does not correspond to a full gauge transformation since we continue to work at fixed gauge $A_0 = 0$. Now, after change (15) there is an electric field whose radial component takes the form

$$\mathcal{F}_{0r} = \frac{1}{e}\frac{d\dot{\lambda}}{dr} \tag{16}$$

with $\dot{\lambda} = (d/dt)\lambda$ satisfying the equation

$$\Delta\dot{\lambda} = \frac{2}{r^2}\dot{\lambda}K^2 \tag{17}$$

In particular, for

$$\lambda(r,t) = \alpha(t)\frac{J}{Mr} \tag{18}$$

(eq.(17) then being satisfied) one obtains the same radial component for the electric field as that arising from the static Julia-Zee dyon, eq.(12), provided one choses $\alpha(t) = -Mt$. Now, Lagrangian (4) becomes, after transformation (15)

$$\mathcal{L} = \frac{1}{2e^2}\partial^i(\dot{\lambda}\partial_i\dot{\lambda}) + M_{mon} \tag{19}$$

where M_{mon} is the 't Hooft-Polyakov monopole mass. At the quantum level, when working in the monopole sector, the term M_{mon} should be substracted in order to normalize the zero-point energy so that the monopole energy is equal to zero [15].

Concerning the CP violating Lagrangian (3) it becomes

$$\Delta\mathcal{L} = -\frac{\theta}{8\pi^2}\frac{1}{r^2}\frac{d}{dr}[\dot{\lambda}(K^2(r)-1)] \tag{20}$$

Using eqs.(19)-(20) and treating λ as a collective coordinate one can reobtain Witten's result (1) at zero temperature. We shall not perform this analysis here but directly consider the case $T \neq 0$.

Finite temperature calculations are carried out as usual by compactifying the (Euclidean) "time" variable into the range $0 \leq \tau \leq \beta = 1/T$ (in our units, $\hbar = c = k = 1$). Periodic boundary conditions (in "time") have to be used for gauge and Higgs field. In particular [18]

$$\vec{A}_i(\vec{x},\beta) = \vec{A}_i^{\Lambda}(\vec{x},0) \tag{21}$$

That is, periodicity modulo τ-independent transformations $\exp[i\Lambda(\vec{x})]$ with $\Lambda(\infty) = 0$ is imposed. Here, we have defined

$$\vec{A}_i^{\Lambda}(\vec{x},0) = \exp[i\Lambda(\vec{x})]\vec{A}_i(\vec{x},0)\exp[-i\Lambda(\vec{x})] + \frac{i}{e}\exp[i\Lambda(\vec{x})]\partial_i\exp[-i\Lambda(\vec{x})] \tag{22}$$

The correct partition function at finite temperature is then given by

$$\mathcal{Z} = \int_{\Lambda(\infty)=0} D\Lambda \int_{\vec{A}_i(\vec{x},\beta)=\vec{A}_i^{\Lambda}(\vec{x},0)} D\vec{A}_i\, D\vec{\phi}\exp\left(-\int_0^{\beta} d\tau\int d^3xL\right), \tag{23}$$

with L the Euclidean version of Lagrangian (6). The Λ integration takes into account the imposition of Gauss law on physical states within the path-integral framework. As explained above, the A_i integration covers all fields periodic up to a twist. Concerning the ϕ integration, is should be performed over periodic field configurations (see ref.[18] for a thoroughful discussion on the definition of the partition function as in eq.(23)).

To see how a careful handling of the dyon degree of freedom implies its charge quantization according to eq.(1) also at finite temperature, we follow the same collective coordinate treatment leading to charge quantization at $T = 0$ [2]. To this end we write the path-integral variables in the form

$$
\begin{aligned}
A_i(\vec{x},\tau) &= A_i^\lambda(\vec{x},\tau) + a_i(\vec{x},\tau) \\
\phi(\vec{x},\tau) &= \phi^{mon}(\vec{x}) + \varphi(\vec{x},\tau)
\end{aligned}
\tag{24}
$$

with $A_i^\lambda(\vec{x},\tau)$ the U-transformed 't Hooft-Polyakov monopole solution,

$$
A_i^\lambda(\vec{x},\tau) = U[\lambda]A_i^{mon}(\vec{x})U^{-1}[\lambda] + \frac{i}{e}U[\lambda]\partial_i U^{-1}[\lambda] \ , \tag{25}
$$

$$
U[\lambda] = \exp[i\lambda(r,\tau)\vec{\Omega}.\vec{t}] \tag{26}
$$

(Collective coordinates associated to non-gauge symmetries will not be considered in what follows since they do not play any role in the charge quantization problem). One has explicitely,

$$
\begin{aligned}
e\vec{A}_i^\lambda &= (\partial_i\lambda)\vec{\Omega} + K\sin\lambda\partial_i\vec{\Omega} + (K\cos\lambda - 1)\vec{\Omega}\wedge\partial_i\vec{\Omega} \\
-ie\vec{F}_{0i}^\lambda &= (\partial_i\dot{\lambda})\vec{\Omega} + \dot{\lambda}K\cos\lambda\partial_i\vec{\Omega} - \dot{\lambda}K\sin\lambda\vec{\Omega}\wedge\partial_i\vec{\Omega} \\
e\vec{F}_{ij}^\lambda &= (K^2-1)\partial_i\vec{\Omega}\wedge\partial_j\vec{\Omega} + \sin\lambda(\partial_i K\partial_j\vec{\Omega} - \partial_j K\partial_i\vec{\Omega}) \\
&\quad + \cos\lambda(\partial_i K\vec{\Omega}\wedge\partial_j\vec{\Omega} - \partial_j K\vec{\Omega}\wedge\partial_i\vec{\Omega})
\end{aligned}
\tag{27}
$$

with now $\dot{\lambda} = (d/d\tau)\lambda$.

In terms of the new variables the partition function reads

$$
Z = \int D\Lambda \int Da_i D\varphi D\lambda \Delta \exp(-\int_0^\beta d\tau \int d^3x L[A^{mon},\phi^{mon},a_i,\varphi,\lambda]) \ , \tag{28}
$$

With Δ the Jacobian associated with the change to the new integration variables. Now, using eqs.(19)-(20) one has

$$
Z = Z_d \times Z_f \tag{29}
$$

where Z_d corresponds to the path-integral over the collective coordinate λ accounting for the dyon's degree of freedom and is obtained using eq.(27) as

$$
\begin{aligned}
Z_d &= \int D\lambda \exp\left(-\frac{2\pi}{e^2}\int_0^\beta d\tau \int_0^\infty r^2 dr \partial^i(\dot{\lambda}\partial_i\dot{\lambda}) \right. \\
&\quad \left. -i\frac{\theta}{2\pi}\int_0^\beta d\tau \int_0^\infty dr \frac{d}{dr}[(K^2-1)\dot{\lambda}]\right)
\end{aligned}
\tag{30}
$$

(We have not explicitely written the Λ integration which, as we shall see below, will be taken into account by giving a precise meaning to the λ integration). Concerning Z_f, it includes the contributions coming from the classical action and the integration over fluctuations a_i and φ not relevant for the analysis of the dyon charge.

In the $T = 0$ case [2] one can identify the Minkowskian version of Z_d, Z_d^M, with the partition function for a one dimensional Coulomb system with a position-dependent coupling (proportional to r^2). The quantum analysis of such a system leads to the usual spectrum of dyon states [13]-[14] with charge given by formula (1) [1]. Alternatively, one can relate Z_d^M with the partition function for a quantum rotator [17] and and also infer dyon charge quantization from the energy spectrum of the rotator [19].

To extend this analysis to the case of finite temperature, let us start by noting that, as in the $T = 0$ case [2], it will be sufficient to consider the collective coordinate λ in the form

$$\lambda = \frac{J(r)}{Mr}\alpha(\tau) \tag{31}$$

with $J(r)$ giving the (classical) electric potential of a Julia-Zee dyon (see eqs.(12),(18)) and $\alpha(\tau)$ describing quantum effects at finite temperature.

In order to unravel the angular character of α let us come back to condition (21) that A_i^λ should also satisfy. Since we have just considered a spherically symmetric collective coordinate λ in the form (31), the allowed transformations for A_i^λ should take the form $\Lambda(\vec{x}) = \Lambda(r)\vec{\Omega}.\vec{t}$ with $\Lambda(r = \infty) = 0$. Explicitely one gets

$$\begin{aligned} (\vec{A}^\lambda)_i^\Lambda(\vec{x}, 0) \;\; &= \;\; \partial_i\left(\lambda(r, 0) + \Lambda(r)\right) + K\sin(\lambda(r, 0) + \Lambda(r))\partial_i\vec{\Omega} + \\ &\quad\;\; \left(K\cos(\lambda(r, 0) + \Lambda(r)) - 1\right)\vec{\Omega} \wedge \partial_i\vec{\Omega} \end{aligned} \tag{32}$$

$$\vec{A}_i^\lambda(\vec{x}, \beta) = \partial_i\lambda(r, \beta) + K\sin\lambda(r, \beta)\partial_i\vec{\Omega} + \left(K\cos\lambda(r, \beta) - 1\right)\vec{\Omega} \wedge \partial_i\vec{\Omega} \tag{33}$$

The "periodicity" condition for A_i^λ thus leads to

$$\left(\alpha(0) - \alpha(\beta)\right)\frac{J(r)}{Mr} + \Lambda(r) = 2\pi n, \quad n \in \mathcal{Z} \tag{34}$$

Now, consistency of this condition at at $r \to \infty$ implies

$$\alpha(0) = \alpha(\beta) + 2\pi n \tag{35}$$

$$\Lambda_n(r) = 2\pi n(1 - \frac{J(r)}{Mr}) \tag{36}$$

We can then rewrite partition function Z_d in terms of the angular variable $\alpha(\tau)$ in the form

$$Z_d = \int D\alpha \exp[-\int_0^\beta d\tau(\frac{\mathcal{J}}{2}\dot{\alpha}^2 - \frac{i\theta}{2\pi}\dot{\alpha})] \tag{37}$$

with $\mathcal{J} = -4\pi b/e^2 M$. In eq.(37) it is understood that the boundary condition (35) is taken into account so that in fact the α-integration covers the different "n-sectors" implied by the Λ integration included in the definition of the finite temperature partition function (23). Note that one can easily relate $\mathcal{J}$ with the dyon electric charge by using eqs.(24)-(31):

$$\frac{Q_{el}}{e} = \frac{1}{e|\vec{\phi}_0|}\int d^3x \partial^i(\vec{\phi}.\vec{F}^{\lambda}_{0i}) = i\mathcal{J}\dot{\alpha} \tag{38}$$

We recognize in eq.(37) the path-integral expression for the transition amplitude of a (modified) rigid rotator evolving (in Euclidean "time" τ) from an "initial" state $\alpha' = \alpha(0)$ to a "final" state $\alpha'' = \alpha(\beta)$ modulo 2π which should be identified with α' according to condition (35). Now, the spectrum of such a rotator can be derived without leaving the path-integral framework as follows [20]. Consider that the "time" interval is $[0, \beta]$ and first solve the classical equations of motion for α as

$$\alpha^{(n)}_{sol}(\tau) = \alpha' + \frac{\tau}{\beta}(\alpha'' - \alpha' + 2\pi n), \tag{39}$$

One then writes the path-integral variable in each "n-sector" in the form

$$\alpha(\tau) = \alpha^{(n)}_{sol}(\tau) + u(\tau) \tag{40}$$

so that Z_d can be written as

$$Z_d = Z_d[\alpha', \alpha'] \tag{41}$$

with

$$Z_d[\alpha', \alpha''] = \mathcal{N}(\beta)\sum_{n=-\infty}^{\infty}\exp[-\frac{\mathcal{J}}{2\beta}(\alpha''-\alpha'+2\pi n)^2 + i\frac{\theta}{2\pi}(\alpha''-\alpha'+2\pi n)] \tag{42}$$

Here $\mathcal{N}(\beta)$ gives a normalization factor arising from integration over fluctuations around the classical trajectories, $\mathcal{N}(\beta) = \sqrt{2\pi\mathcal{J}/\beta}$. One now proceeds to a Fourier expansion

$$Z_d[\alpha', \alpha''] = \sum_{l=-\infty}^{\infty} \exp[il(\alpha'' - \alpha')] \exp(-\beta E_l) \tag{43}$$

with

$$\begin{aligned} \exp(-\beta E_l) \quad &= \sqrt{\frac{\mathcal{J}}{2\pi\beta}} \int_0^{2\pi} d\alpha \exp(-il\alpha) \times \\ &\sum_{n=-\infty}^{\infty} \exp[-\frac{\mathcal{J}}{2\beta}(\alpha + 2\pi n)^2 + i\frac{\theta}{2\pi}(\alpha + 2\pi n)] \end{aligned} \tag{44}$$

Inverting sum and integration and making the shift $\alpha + 2\pi n \to \alpha$ one finally gets

$$\exp[-\beta E_l] = \exp[-\frac{\beta(l - \theta/2\pi)^2}{2\mathcal{J}}] \tag{45}$$

thus giving for the energy levels of the rotator the result

$$E_l = \frac{1}{2\mathcal{J}}(l - \theta/2\pi)^2 \tag{46}$$

We can go further in the analogy with a quantum rigid rotator and evaluate from the Euclidean Lagrangian in partition function (37) the (angular) momentum p_α

$$p_\alpha \equiv i\frac{\partial L}{\partial \dot{\alpha}} = i\mathcal{J}\dot{\alpha} + \frac{\theta}{2\pi} = \frac{Q_{el}}{e} + \frac{\theta}{2\pi} \tag{47}$$

On the other hand, eq.(46) shows that p_α is quantized according to $p_\alpha = l$. From this one finally has the dyon charge quantization condition

$$Q_{el} = e(l - \theta/2\pi), \quad l \in \mathcal{Z} \tag{48}$$

One arrives in this way to Witten formula (1) for the dyon charge, which thus remains unchanged at finite temperature. To corroborate this result obtained just by using the quantum-mechanics analogy, one can directly evaluate

$$< i\mathcal{J}\dot{\alpha} + \frac{\theta}{2\pi} >= \frac{1}{Z_d} \int D\alpha (i\mathcal{J}\dot{\alpha} + \frac{\theta}{2\pi}) \exp[-\int_0^\beta d\tau (\frac{\mathcal{J}}{2}\dot{\alpha}^2 - i\frac{\theta}{2\pi}\dot{\alpha})] \tag{49}$$

Proceeding as before, we find

$$
\begin{aligned}
< i\mathcal{J}\dot{\alpha} + \frac{\theta}{2\pi} > &= < \frac{Q_{el}}{e} + \frac{\theta}{2\pi} > \\
&= \frac{1}{Z_d} \sum_l l \, \exp(-\beta E_l)
\end{aligned} \tag{50}
$$

which again leads to quantization of p_α as an integer l and thus to eq.(48).

In summary, our analysis indicates that the formula (1) found in [1] for the electric charge of a 't Hooft-Polyakov magnetic monopole in CP non-conserving spontaneously broken gauge theories remains unchanged at finite temperature. This result should be compared with that obtained in [7] for a fixed Dirac monopole in an electron-positron plasma at temperature T. In this last investigation the induced charge does depend smoothly on the temperature and on the fermion mass m through the dimensionless combination m/T (being their monopole an Abelian Dirac one, there is no other mass parameter to enter in a dimensionless variable describing the temperature dependence). This behavior can be related, as noted in [7] to perturbative analysis leading to a temperature dependent coefficient for the Chern-Simons effective action in 3-dimensional fermionic theories. However, it was shown in [8]-[10] that perturbative approaches yielding to this last result are inconsistent with the requirement of gauge-invariance. Moreover, already at $T = 0$ the fermion-Dirac monopole system has shown to lead to different monopole quantum numbers according to different boundary condition for the Dirac wave function at the location of the monopole. As it is the case for Witten analysis [1], our treatment stems from more general grounds and applies to extended 't Hooft- Polyakov monopoles in spontaneously broken gauge theories where temperature dependence could in principle arise in terms of a dimensionless variable, for example the combination m_W/T (with m_W the mass of the W vector boson). However, because of gauge-invariance and topological considerations, already at the root of eq.(1) at $T = 0$, we have shown that the dyon electric charge remains unchanged at finite temperature.

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